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Minimum Lift-Drag Ratio Required for Global Landing Coverage

Richard A. Wallace* and William A. Gray†
Boeing Company, Seattle, Wash.

A PRINCIPAL advantage of lifting re-entry vehicles is the lateral range obtained by turning during atmospheric flight. With sufficient lift-drag ratio (L/D), a re-entry glider can land at any desired point on the globe after waiting no longer than one circumnavigation to de-orbit. Higher L/D would not increase landing site selection capability. Therefore, the lowest L/D that provides global coverage might be considered a maximum design goal for lifting re-entry vehicles. As described below, an L/D of 3.6 is found sufficient to land a re-entry glider anywhere on earth, if a particular bank-angle schedule is used.

Determination of this minimum L/D is complicated because the landing footprint is strongly influenced by the choice of enroute bank-angle variation. Thus, the problem becomes one of optimizing the bank-angle schedule for minimum L/D with a terminal constraint for global-landing coverage. Although this problem is amenable to a direct mathematical approach,[1] the procedure chosen was a cut-and-try method guided by past results of such mathematical procedures[2] and past experience with turning trajectories. This procedure gives reasonable results and an insight into the influence of bank-angle schedules on L/D values required for global coverage.

The minimum L/D for global coverage was found for several different bank schedules by plotting the maximum lateral range reached vs L/D and extrapolating to the necessary range for complete global coverage. In Fig. 1 the necessary lateral range is described simply as the ability to reach the North Pole after de-orbiting along the equator. Figure 2 illustrates the four bank-angle schedules that were studied.

In simplified treatments of this problem, a bank-angle of 45° is considered close to optimum. This is satisfactory for L/D less than or equal to 2, but it is unsatisfactory for large L/D and large lateral range.

The bank-angle schedules that depend on heading (Fig. 2a) were found to be the most effective of those tried in accomplishing large lateral range with minimum L/D. Because large bank angles produce quick heading change but also reduce flight range, it seems reasonable to expect that the best bank schedule will incorporate large bank angle to generate initial heading change followed by more moderate bank angle to conserve range.

These trends are confirmed by turn optimization studies[2] for vehicles capable of lateral range on the order of 2000 naut miles (L/D = 2). Gains of around 10% in lateral range are obtained by using an optimum diminishing bank-angle schedule rather than simple 45° bank turns, which appear best in simplified analyses.[3] For global coverage, however, much larger gains are available. Contrast the results in Fig. 1 for 45° bank schedules D or $C2$ with those for diminishing bank-angle schedule $A2$, and note the widening difference in lateral range (as indicated by closer approach to the North Pole) as L/D increases. The superior performance with diminishing bank schedules materially reduces the L/D needed to reach the pole.

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* Assocate Engineer, Aero-Space Division.

† Now Senior Engineer, North American Aviation Inc., Downey, Calif. Member AIAA.

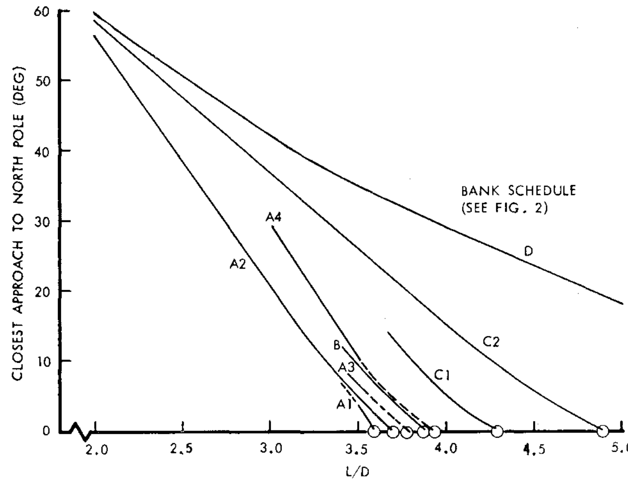

Fig. 1 Necessary lateral range described simply as the ability to reach the North Pole after de-orbiting along the equator.

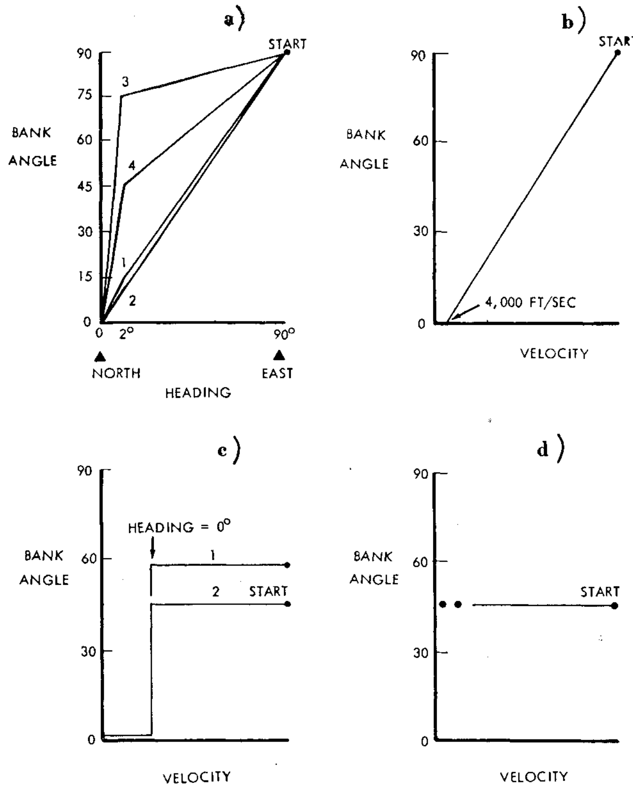

Fig. 2 a) Bank angle varied linearly with heading at two rates: low-rate reduction until the heading equals 2°, then rapid reduction in bank angle as the heading goes to zero (due north). b) Bank angle varied linearly with velocity. c) Bank angle kept constant until heading reaches zero (due north); thereafter the bank angle is kept zero. d) Bank angle kept constant at 45° throughout the flight.

In arriving at the results of this study, entry and vehicle conditions were specified. Generality is not lost, however, since Slye[3] shows that a turn is relatively insensitive to variations in re-entry lift loading (W/SC_L) and initial flight conditions if it is initiated at speeds equal to or slightly in excess of the original orbital velocity. Therefore, the results of this study may be considered applicable for any reasonable W/SC_L and for return from any *low-altitude* orbit.

Entry trajectories were computed by integrating the complete equations of motion on an IBM 7090 computer. Characteristics selected were as follows. De-orbit was accomplished with an impulse of 95 fps from an equatorial, circular orbit at an 80-naut-mile altitude. A nonrotating spherical earth model was used and a value of 100 psf specified for W/SC_L.

An L/D of 3.6 was found to be adequate for global coverage when advantage is taken of a varying bank-angle schedule. With a constant 45° bank angle (schedule D), an L/D of about 6.5 would be required.

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Method of Determining Saturated Liquid and Saturated Vapor Entropy

Malcolm A. Walker*

Aerojet-General Corporation, Sacramento, Calif.

Nomenclature

C_{SL} = specific heat under saturated liquid conditions
C_p = specific heat at constant pressure
H = enthalpy
ΔH_{LV} = latent heat of vaporization
P = pressure
S = entropy
ΔS_{LV} = latent entropy of vaporization
T = absolute temperature
T_c = critical temperature
V = specific volume

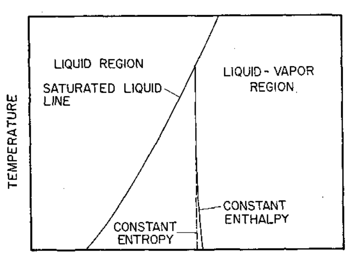

Fig. 1 Saturated liquid region.

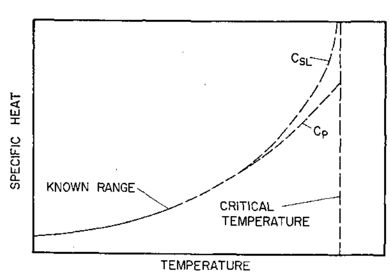

Fig. 2 Extrapolation of specific heat.

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* Project Engineer, Advanced Development Division, Liquid Rocket Plant.

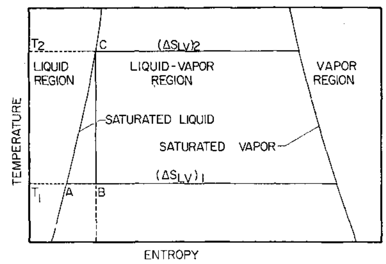

Fig. 3 Construction of temperature-entropy diagram.

Subscripts

H = constant enthalpy process
S = constant entropy process
SL = saturated liquid conditions

I. Introduction

The present lack of thermodynamic data available for all but the most common liquid propellants makes difficult the analysis of physical processes, such as the expansion of saturated liquids in flow-control nozzles. The following method is presented as a means of determining entropies in the liquid-vapor region, for temperatures not exceeding the critical point. The resulting temperature-entropy diagrams have been found to be in excellent agreement with accepted diagrams and with those computed by far more rigorous techniques.

II. Analysis

It can be shown from basic thermodynamics that[1]

$$dH = TdS + VdP \tag{1}$$

The relative magnitude of the VdP term for liquids is so small (less than 0.5% of dH for saturated liquid water at 150 psia and 358°F) that it may be neglected with small error, and the following is obtained:

$$dH = TdS \tag{2}$$

From this it follows that, if either entropy or enthalpy is constant, its derivative will reduce to zero, and the other must also be constant, and so it may be written as

$$(\partial H)_S = 0 \tag{3}$$

$$(\partial S)_H = 0 \tag{3a}$$

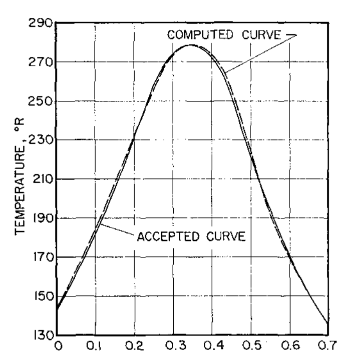

Fig. 4 Comparison of calculated and accepted temperature - entropy diagrams for oxygen.

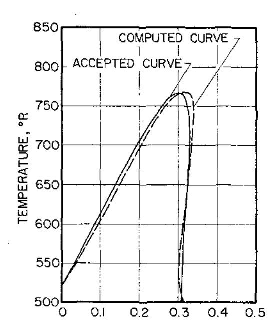

Fig. 5 Comparison of calculated and accepted temperature-entropy diagrams for *n*-butane.